Midterm Exam Calculus 2

12 March 2015, 14:00-16:00

university of groningen

The exam consists of 4 problems. You have 120 minutes to answer the questions. You can achieve 100 points which includes a bonus of 10 points.

1. [**5+10+10 Points.**]

   Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

   $$f(x,y) = \begin{cases} \frac{3x^2y-y^3}{x^2+y^2} & \text{if } \langle x,y\rangle \neq \langle 0,0\rangle \\ 0 & \text{if } \langle x,y\rangle = \langle 0,0\rangle \end{cases}.$$

   (a) Is f continuous at $\langle x,y\rangle = \langle 0,0\rangle$? Justify your answer.

   (b) Compute the partial derivative of f at $\langle x,y\rangle \neq \langle 0,0\rangle$.

   (c) Use the definition of partial derivatives to determine the partial derivatives of f at $\langle x,y\rangle = \langle 0,0\rangle$.

2. [**10+5+5 Points.**]

   Consider the curve parametrized by $\mathbf{r} : [1,4] \to \mathbb{R}^3$ with

   $$\mathbf{r}(t) = \ln t\,\mathbf{i} + \frac{t^2}{2}\,\mathbf{j} + \sqrt{2}\,t\,\mathbf{k}.$$

   (a) Determine the parametrization of the curve by arclength.

   (b) For each point on the curve, determine a unit tangent vector.

   (c) At each point on the curve, determine the curvature of the curve.

3. [**10+15 Points.**]

   Consider the ellipsoid

   $$3x^2 + 2y^2 + z^2 = 6.$$

   (a) Determine the tangent plane of the ellipsoid at the point $\langle x,y,z\rangle = \langle 1,1,1\rangle$.

   (b) Use the method of Lagrange multipliers to find the largest sphere centered at the origin which is inscribed in (i.e. contained in the region enclosed by) the ellipsoid.

4. [**20 Points.**]

   Integrate the function

   $$f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}\,\mathrm{e}^{x^2+y^2+z^2}$$

   over the region $W \subset \mathbb{R}^3$ enclosed by the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 2$.